(iii) Force of charges in the insulated sphere :-

From the defⁿ of coulomb's force we know that

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

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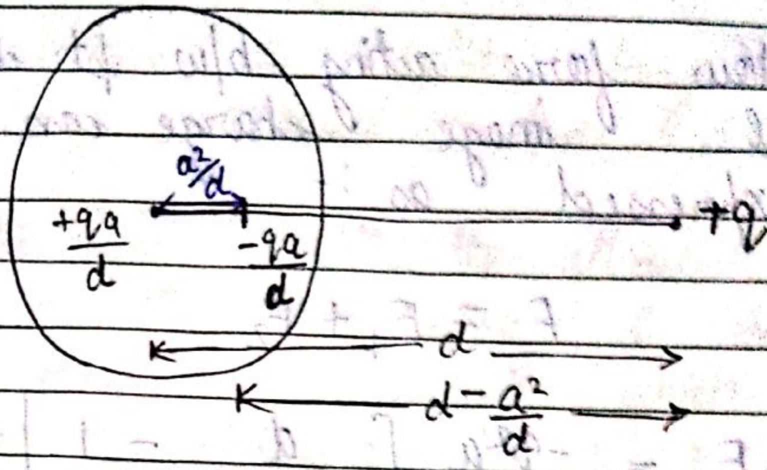

From fig the force b/w the charge $\left(+\frac{qa}{d}\right)$ & $+q$ can be expressed as :-

$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{\left(+\frac{qa}{d}\right)(+q)}{d^2}$$

$$F_1 = +\frac{1}{4\pi\epsilon_0} \frac{q^2 a}{d^3}$$

Now the force b/w the charges $\left(-\frac{qa}{d}\right)$ & $+q$ can be calculated as :-

$$F_2 = \frac{1}{4\pi\epsilon_0} \frac{\left(-\frac{qa}{d}\right)(+q)}{\left(d - a^2/d\right)^2} \Rightarrow (d - a^2/d)^2$$

$$F_2 = -\frac{1}{4\pi\epsilon_0} \frac{q^2 a/d}{(d^2-a^2)^2/d^2}$$

$$F_2 = -\frac{1}{4\pi\epsilon_0} \frac{q^2 a \cdot d}{(d^2-a^2)^2}$$

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Now force acting b/w pt. charge & image charge can be expressed as :-

$$F = F_1 + F_2$$

$$F = \frac{-q^2 a}{4\pi\epsilon_0}\left[\frac{d}{(d^2-a^2)^2} - \frac{1}{d^3}\right]$$

$$= \frac{-q^2 a}{4\pi\epsilon_0}\left[\frac{d^4 - d^4 - a^4 + 2a^2d^2}{d^3(d^2-a^2)^2}\right]$$

$$= \frac{-q^2 a}{4\pi\epsilon_0} \times a^2 \left[\frac{2d^2 - a^2}{d^3(d^2-a^2)^2}\right]$$

$$\boxed{F = \frac{-q^2 a^3}{4\pi\epsilon_0}\left[\frac{(2d^2-a^2)}{d^3(d^2-a^2)^2}\right]}$$

Since $d > a$ so $F = -ve$.
It means the force acting b/w the pt. charge & image charge is attractive in nature.
Now two cases may arises :-

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Case 1 :- When the point charge is at large distance :- i.e., $d >>> a$

In such cond^n the attractive force b/w the pt. charge & image charge is changes which can be expressed as :-

$$F = \frac{-q^2}{4\pi\epsilon_0}\left[\frac{a^3(2d^2)}{d^3\, d^4}\right]$$

$$F = \frac{-q^2}{4\pi\epsilon_0}\, a^3\, \frac{2}{d^5}$$

i.e.,
$$\boxed{F \propto \frac{-1}{d^5}}$$

i.e., the force obeys the inverse fifth power law.

Case II → When the pt. charge is nearer to the sphere i.e.

Let us consider a ~~direction~~ distance 'x' which is very less than the radius of the sphere.

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Now, $d = a + x$

then

$$F = \frac{-q^2}{4\pi\epsilon_0}\left[\frac{a^3}{(a+x)^3}\left\{\frac{2(a+x)^2 - a^2}{(a+x)^2 - a^2}\right\}^2\right]$$

$$F = \frac{-q^2}{4\pi\epsilon_0}\left[\frac{a^3}{(a+x)^3}\left\{\frac{a^2+2x^2+4ax}{(x^2+2ax)^2}\right\}\right]$$

when, $a >>> x$ then above eqn reduce in the form of :-

$$F = \frac{-q^2}{4\pi\epsilon_0}\left[\frac{a^3}{a^3}\left\{\frac{a^2+4ax}{4a^2x^2}\right\}\right]$$

$$F = \frac{-q^2}{4\pi\epsilon_0}\,\frac{1}{4x^2}$$

i.e., the force is ~~l'ly~~ proportional to the inverse square of the distance from the ~~sphere~~ surface